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PRIME RING WITH SYMMETRIC SKEW 4- REVERSE DERIVATIONS

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ABSTRACT

In this paper we introduce the notion of symmetric skew 4-reverse derivation of prime ring and we consider R be a non commutative 2,3- torsion free ring , I be a non zero two sided ideal of R. Suppose α is an anti automorphism of R, and $D: R \times R \times R \times R \to R$ be a symmetric skew 4-reverse derivation associated with the anti automorphism α. If f is trace of D such that $[f(x), \alpha(x)] = 0$, for all $x \in I$, then $D = 0$.

Key words: *prime ring, Reverse Derivation, Symmetric Skew 3- derivation, Symmetric Skew 4- reverse derivation and Anti automorphism.*

INTRODUCTION

Bresar and Vukman [3] have introduced the notion of a reverse derivations and Samman and Alyamani [9] have studied some properties of semi prime rings with reverse derivations. AjdaFosner [1] have introduced the notation of symmetric skew 3- derivations of prime or semi prime rings and proved that under certain conditions a prime ring with a non zero symmetric skew 3-derivation has to be commutative. The study of centralizing and commuting mappings on prime rings was initiated by the result of Posner [8] which states that the existence of a non zero centralizing derivation on a prime ring implies that the ring has to be commutative. Vukman [10, 11] investigated symmetric bi derivation on prime and semi prime rings in connection with centralizing mappings. C.JayaSubba Reddy *et al.* [5] has studied prime ring with symmetric skew 3- reverse derivations. Faiza Shujat and Abuzaid Ansari [4] has studied symmetric skew 4-derivations on prime rings. In this paper we proved that under certain conditions a prime ring with a non zero symmetric skew 4-reverse derivation has to be commutative.

PRELIMINARIES

Throughout the paper, R will represent a ring with a center Z and α an anti automorphism of R. Let $n \geq 2$ be an integer. A ring R is said to be n-torsion free if for $x \in R, nx = 0$ implies $= 0$. For all $x, y \in R$ the symbol $[x, y]$ will denote the commutator$xy - yx$. Recall that a ring R is semi prime if $xRx = 0$,implies that $x = 0$.An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. An additive map $d: R \to R$ is called reverse derivation if $d(xy) = d(y)x + yd(x)$, for all $x, y \in R$ and it is called a skew derivation (α -derivation) of R associated with the automorphism or anti automorphism α if $d(xy) = d(x)y + \alpha(x)d(y)$, for all $x, y \in R$, and it is called a skew reverse derivation ($\alpha -$ reverse derivation) of R associated with auto morphism or anti automorphism α if $d(xy) = xd(y) + \alpha(y)d(x)$, for all $x, y \in R$.

Before starting our main theorem, let us gives some basic definations and well known results which we will need in our further investigation.

Let D be a symmetric 3-additive map of R, then obviously

$$D(-x, y, z) = -D(x, y, z), \text{ for all } x, y, z \in R \quad (1)$$

Namely, for all $y, z \in R$, the map $D(., y, z): R \to R$ is endo morphissm of the additive group of R.

The map $f: R \to R$ defined by $f(x) = D(x, x, x, x)$, $x \in R$ is called trace of D

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Note that f is not additive on R. But for all $x, y \in R$, we

$$f(y+x) = f(y) + f(x) + 3D(y,x,x) + 3D(x,y,y)$$

Recall also that by (1), f is odd function

More precisely, for all $x, y, z, u, v, w \in R$, we have

$$D(xu,y,z) = xD(u,y,z) + \alpha(u)D(x,y,z),$$
$$D(x,yv,z) = yD(x,v,z) + \alpha(v)D(x,y,z),$$
$$D(x,y,zw) = zD(u,y,w) + \alpha(w)D(x,y,z).$$

Of course, if D is symmetric, then the above three relations are equivalent to each other.

MAIN RESULT

Theorem: Let R be a 2,3 −torsion free non commutative prime ring and I be a nonzero ideal of R.Suppose α is an anti automorphism of R and $D: R^4 \to R$ is a symmetric skew 4- reverse derivation associated with α. If f is trace of D such that $[f(x), \alpha(x)] = 0$, for all $x \in I$, then $D = 0$.

Proof: Let $[f(x), \alpha(x)] = 0$, for all $x \in I$. (2)

Linearization of (2) yields that

$[f(x+y), \alpha(x+y)] = 0$

$[f(x) + 4D(x,x,x,y) + 6D(x,x,y,y) + 4D(x,y,y,y) + f(y), \alpha(x)] = 0$

$[f(x),\alpha(x)] + 4[D(x,x,x,y),\alpha(x)] + 6[D(x,x,y,y),\alpha(x)] + 4[D(x,y,y,y),\alpha(x)] + [f(y),\alpha(x)] + [f(x),\alpha(y)]$
$+4[D(x,x,x,y),\alpha(y)] + 6[D(x,x,y,y),\alpha(y)] + 4[D(x,y,y,y),\alpha(y)] + [f(y),\alpha(y)] = 0$, for all $x \in I$. (3)

From (2) & (3), we get

$4[D(x,x,x,y),\alpha(x)] + 6[D(x,x,y,y),\alpha(x)] + 4[D(x,y,y,y),\alpha(x)] + [f(y),\alpha(x)] + [f(x),\alpha(y)]$
$+4[D(x,x,x,y),\alpha(y)] + 6[D(x,x,y,y),\alpha(y)] + 4[D(x,y,y,y),\alpha(y)] = 0$, for all $x \in I$. (4)

Replacing y by $-y$ in (4) we find

$-4[D(x,x,x,y),\alpha(x)] + 6[D(x,x,y,y),\alpha(x)] - 4[D(x,y,y,y),\alpha(x)] + [f(y),\alpha(x)] - [f(x),\alpha(y)]$
$+4[D(x,x,x,y),\alpha(y)] - 6[D(x,x,y,y),\alpha(y)] + 4[D(x,y,y,y),\alpha(y)] = 0$, for all $x \in I$. (5)

Comparing (4) and (5) and using 2-torsion freeness of R we get

$4[D(x,x,x,y),\alpha(x)] + 4[D(x,y,y,y),\alpha(x)] + [f(x),\alpha(y)] + 6[D(x,x,y,y),\alpha(y)] = 0$,for all $x \in I$. (6)

Substitute $y+z$ for y in (6) and use (6) to get

$$4[D(x,x,x,y+z),\alpha(x)] + 4[D(x,y+z,y+z,y+z),\alpha(x)] + [f(x),\alpha(y+z)] + 6[D(x,x,y+z,y+z),\alpha(y+z)] = 0$$

$$\begin{aligned}4[D(x,x,x,y),\alpha(x)] &+ 4[D(x,x,x,y),\alpha(x)] + 4[D(x,y,y,y),\alpha(x)] + 4[D(x,y,y,z),\alpha(x)] + 4[D(x,y,z,y),\alpha(x)]\\ &+ 4[D(x,y,z,z),\alpha(x)] + 4[D(x,z,y,y),\alpha(x)] + 4[D(x,z,y,z),\alpha(x)] + 4[D(x,z,z,y),\alpha(x)]\\ &+ 4[D(x,z,z,z),\alpha(x)] + [f(x),\alpha(y)] + [f(x),\alpha(z)] + 6[D(x,x,y,y),\alpha(y)]\\ &+ 6[D(x,x,y,z),\alpha(y)] + 6[D(x,x,z,y),\alpha(y)] + 6[D(x,x,z,z),\alpha(y)] + 6[D(x,x,y,y),\alpha(z)]\\ &+ 6[D(x,x,y,z),\alpha(z)] + 6[D(x,x,z,y),\alpha(z)] + 6[D(x,x,z,z),\alpha(z)] = 0\end{aligned}$$

$12[D(x,x,x,y),\alpha(x)] + 12[D(x,z,y,y),\alpha(x)] + [D(x,x,y,z),\alpha(y)] + 6[D(x,x,z,z),\alpha(y)]$
$+6[D(x,x,y,y),\alpha(z)] + 12[D(x,x,y,z),\alpha(z)] = 0$, for all $x,y,z \in I$ (7)

Replacing z in $-z$ in (7) and compare with (7) we obtain

$$\begin{aligned}-12[D(x,x,x,y),\alpha(x)] &+ 12[D(x,z,y,y),\alpha(x)] + 12[D(x,x,y,z),\alpha(y)] - 6[D(x,x,z,z),\alpha(y)]\\ &+ 6[D(x,x,y,y),\alpha(z)] - 12[D(x,x,y,z),\alpha(z)] = 0\end{aligned}$$

$2(12[D(x,z,y,y),\alpha(x)] + 12[D(x,x,y,z),\alpha(y)] + 6[D(x,x,y,y),\alpha(z)]) = 0$

Using of two torsion free ring we have

$12[D(x,z,y,y),\alpha(x)] + 12[D(x,x,y,z),\alpha(y)] + 6[D(x,x,y,y),\alpha(z)] = 0$, for all $x,y,z \in I$. (8)

Substitute $y+u$ for y in (8) and use (8) we get

$12[D(x,z,y+u,y+u),\alpha(x)] + 12[D(x,x,y+u,z),\alpha(y+u)] + 6[D(x,x,y+u,y+u),\alpha(z)] = 0.$

$12[D(x,z,y,y),\alpha(x)]+12[D(x,z,y,u),\alpha(x)]+12[D(x,z,u,y),\alpha(x)]+12[D(x,z,u,u),\alpha(x)]$
$+12[D(x,x,y,z),\alpha(y)]+12[D(x,x,u,z),\alpha(y)]+12[D(x,x,y,z),\alpha(u)]$
$+12[D(x,x,u,z),\alpha(u)]+6[D(x,x,y,y),\alpha(z)]+6[D(x,x,y,u),\alpha(z)]+6[D(x,x,u,y),\alpha(z)]$
$+6[D(x,x,u,u),\alpha(z)]=0.$

$24[D(x,z,y,u),\alpha(x)]+12[D(x,x,y,z),\alpha(u)]+12[D(x,x,u,z),\alpha(y)]+12[D(x,x,y,u),\alpha(z)]=0,$
for all $x,y,z\in I$. (9)

Since R is 2 and 3-torsion free and replacing y,u by x in (9), we have
$24[D(x,z,x,x),\alpha(x)]+12[D(x,x,x,z),\alpha(x)]+12[D(x,x,x,z),\alpha(x)]+12[D(x,x,x,x),\alpha(z)]=0.$
$48[D(x,x,x,z),\alpha(x)]+12[D(x,x,x,x),\alpha(z)]=0.$
$4[D(x,x,x,z),\alpha(x)]+[f(x),\alpha(z)]=0$, for all $x,z\in I$. (10)

Again replaced z by yz in (10) and using (10) we obtain
$4[D(x,x,x,yz),\alpha(x)]+[f(x),\alpha(yz)]=0$

$4[yD(x,x,x,z)+\alpha(z)D(x,x,x,z),\alpha(x)]+[f(x),\alpha(z)\alpha(y)]=0$

$4y[D(x,x,x,z),\alpha(x)]+4[y,\alpha(x)]D(x,x,x,z)+4\alpha(z)[D(x,x,x,z),\alpha(x)]+4[\alpha(z),\alpha(x)]D(x,x,x,z)$
$+[f(x),\alpha(z)]\alpha(y)+\alpha(z)[f(x),\alpha(y)]=0$

$\alpha(z)(4[D(x,x,x,z),\alpha(x)]+\ [f(x),\alpha(y)]+4y[D(x,x,x,z),\alpha(x)]$
$+4[y,\alpha(x)]D(x,x,x,z)4[\alpha(z),\alpha(x)]D(x,x,x,z)+[f(x),\alpha(z)]\alpha(y)=0$

$4y[D(x,x,x,z),\alpha(x)]+4[y,\alpha(x)]D(x,x,x,z)+4[\alpha(z),\alpha(x)]D(x,x,x,z)+[f(x),\alpha(z)]\alpha(y)=0,$
for all $x,y,z\in I$. (11)

Substitute x for z in (11) and view of (2) we find
$4[y,\alpha(x)]f(x)=0$, for all $x,y\in I$. (12)

Using 2- torsion freeness of R we obtain
$[y,\alpha(x)]f(x)=0$, for all $x,y\in I$.(13)

Substitute zy for y to get
$[zy,\alpha(x)]f(x)=0$

$[z,\alpha(x)]yf(x)+z[y,\alpha(x)]f(x)=0]$

$[z,\alpha(x)]yf(x)=0$, for all $x,y\in I$. (14)

Primeness of R yields that $[z,\alpha(x)]=0$ or $f(x)=0$, for all $x\in I\cap Z(R), z\in I$.

Next we will show that $f(x)=0$, for all $x\in I$.

Let $z\in I\cap Z(R)$ and $x\in I\backslash Z(R)$.

Then $x+z, x-z\in I\ /Z(R)$ and we have
$f(x+z)=f(z)+4D(x,x,x,z)+4D(x,z,z,z)+6D(x,x,z,z)$ (15)
and
$f(x+z)=f(z)-4D(x,x,x,z)-4D(x,z,z,z)+6D(x,x,z,z)$ (16)

Comparing (15) & (16) and using 2- torsion free condition, we get
$f(z)+6D(x,x,z,z)=0.$ (17)

On suitable linearization and using (17) we arrive at $f(x)=0$, for all $x\in I$. Hence we have $D(x,y,z,w)=0$, for all $x,y,z,w\in I$.

Substitute xr for x for all $x\in I, r\in R$ to get
$D(xr,y,z,w)=0$
$xD(r,y,z,w)+\alpha(r)D(x,y,z,w)=0$
$xD(r,y,z,w)=0$ (18)
This implies that $ID(r,y,z,w)=0$, for all $y,z,w\in I, r\in R$.

Since R is prime we obtain $D(r,y,z,w)=0$, for all $y,z,w\in I$, $r\in R$.

Repeating this process until we get $D(r,s,t,p)\in R$.

Hence $D=0$.

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